

Performance Attribution of Money-Weighted Return without Rebalancing to Strategy Allocation

The analysis of the management style of an investment portfolio is typically carried out with performance attribution. The traditional performance attribution model, however, has weaknesses or at least some traits that do not entirely correspond to the behavior of asset managers or to the expectations of investors. In particular, the changes in weights of the different asset classes are entirely attributed to the asset manager although they partly result from their different market tendencies. We suggest a more intuitive tool to assess “What has the asset manager done for us?” We compare the effective performance of the investment with the “do nothing” performance, which would have been achieved if no investment decision had been taken during the time period under scrutiny. Only the active trades of the asset manager influence the active component of the performance, which we call the trading performance. Moreover, this trading performance can be split into a turnover and a selection component. The turnover component reflects the shifts between asset classes executed by the asset manager and the selection component is defined as in the traditional performance attribution model. The trading performance is much more than a performance number; it is a time series of performances clearly showing the influence of each transaction decided by the asset manager; decomposed into a turnover and a selection component. It is therefore possible to determine the contributions of each transaction to performance.

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INTRODUCTION

Traditionally, the analysis of the management style of investment portfolios is carried out with the performance attribution models as described in Brinson and Fachler (1985) and Brinson *et al.* (1986) and numerous later extensions. We start by reviewing the properties of this method that we call the “Brinson model.” We draw attention to some assumptions behind the model that do not entirely correspond to the behavior of asset managers or to the expectations of their clients.

We then present another model of performance attribution of the Money-Weighted Return (MWR), instead of the Time-Weighted Return (TWR) for the Brinson

model.

This model does not take the strategy weights into account. It computes the contribution to the MWR of the investment over the “do nothing” strategy for each transaction individually, decomposing this contribution into a “turnover” and a “selection” component. The result consists of time series of a turnover and selection components, allowing us to discern the transactions or groups of transactions having mostly contributed to one component or the other.

We finish this article by presenting the results of a real life portfolio of a pension fund and analyzing these results.

THE TRADITIONAL “BRINSON MODEL”

The analysis of the asset management is traditionally carried out with the help of performance attribution. The performance attribution decomposes the performance of the investment in a passive and an active component. The passive component assumes an investment replicating exactly the underlying benchmarks and according to the target weights of the investment strategy. The active component of the performance is the difference between the effective performance and the passive performance. This difference is in turn decomposed into an allocation component that results from a manager's weighting of the asset classes differing from the targeted strategic weighting, a security selection component that results from an investment selection within an asset class differing from that investment held in the underlying benchmark, and an interaction component that cannot be directly attributed to a specific pattern of investment. The performance attribution methodology has been initiated by Brinson and Fachler (1985) and Brinson, Hood and Beebower (1986), and has been extended and refined in numerous publications since then.

The performance attribution as in the Brinson model, however, shows several limitations:

1. The Brinson model relies heavily on the investment strategy and the benchmarks, but ignores the latitude which is usually given to the asset manager to deviate from the strategy weights of the asset classes. These weights do not only change because of active decisions of the asset manager but also because of the different market tendencies of the asset classes. The asset manager usually takes advantage of the leeway given to her and does not continuously rebalance the portfolio, even if her views of the different asset classes are neutral concerning their performance expectations. Under the Brinson model, however, “doing nothing” is considered active management. The client has another perception of the active asset management. Each trade is seen as an active decision of the asset manager, if this trade was done for rebalancing purposes, or actively over- and underweighting some asset classes is irrelevant.
2. The Brinson model decomposes the TWR performance of the investment and thus reflects the per-

formance from the point of view of the asset manager. The client point of view of the performance can only be fully taken into account by decomposing the MWR performance as in Akeda (2001), Illmer and Marty (2003) or Armitage and Bagot (2004). They also consider the question, “What has the manager done for me?” as central (Turnovers at the discretion of the asset manager). Considering the TWR performance makes it impossible to compute the contribution to performance of individual transactions, which is only workable with a value-based method.

3. The calculation of performance attribution on a large interval relies on the division of the interval of analysis into small intervals. Unfortunately, the aggregation of the results of the small intervals into the global results of the original interval of analysis has not been solved. Several methods have been suggested, *i.e.*, GRAP (1997), Cariño (1999), Menchero (2000), Davis and Laker (2001), Frongello (2002), and but so far none can be considered as the standard procedure.
4. Next to the “allocation” and the “selection” components of the active asset management there is an “interaction” term that cannot be assigned to either the allocation or the selection behavior of the asset manager because it results from a difference in the allocation and a difference in the selection. This would not be a serious problem if the interaction term was always small compared to the other components. Unfortunately, this is not always the case.
5. The Brinson model necessitates the recalculation of weights for each period in the holdings-based model, and for each transaction in the transaction based model. This means that either the weights of the asset classes are considered constant on rather large time intervals, although they vary with the evolution of market prices, or the process requires a relatively large amount of data.

As a consequence, investors often spend a considerable amount of time and resources analyzing the transactions of the asset manager. Usually this means going through hundreds or even thousands of transactions without knowing what the reason behind the transaction is. This leads the investor to screening only the largest transac-

tions, thus neglecting the choices of the asset manager that resulted in several transactions, sometimes spread over a time span of a few days.

NEW PERFORMANCE ATTRIBUTION MODEL

We now present a new model of performance attribution. The main difference to the Brinson model lies in the definition of the neutral policy. In the Brinson model the neutral policy is the investment strategy, whereas in our new measure the neutral policy is the “do nothing” policy. This means that the asset manager does not intervene even when transactions occur that are direct consequences of the initial holdings. For example, the asset manager does not reinvest the cash received from a dividend payment in the “do nothing” policy. As a corollary of this definition, the neutral return of the investment is the return of the holdings at the beginning of the measurement period: the initial holdings. We then examine each transaction individually and determine whether it has increased or decreased the performance of the initial holdings. This increase or decrease is called the contribution of the transaction to the performance. We then decompose this contribution to performance in a turnover and a selection component. The turnover performance is the component of the contribution due to turnover between investment classes, and the selection component is due to a change of security within the same asset class. We have changed the terminology of classical performance attribution from “allocation” to “turnover,” since in our model we do not recompute the weights of the asset classes and compare them with those of the investment strategy. We are just considering the changes in those weights due to the transaction. Let us make clear that by performance we mean the IRR of the investment over the measurement interval. The result is a time series of returns starting with the performance of the initial holdings at the start of the measurement interval, and ending with the IRR of the investment at the end of the measurement interval.

Each transaction that is not a direct consequence of the initial holdings is considered an “active” decision of the asset manager that influences the performance of the original investment (initial holdings), positively or negatively. This is totally independent of the weights of the asset classes in the investment strategy. The time series of contributions to performance allows to distinguish the transactions or groups of transactions that con-

tributed most to the final IRR. The client can analyze the trades of the asset manager with the utmost precision with a measure that reconciles to the IRR, *i.e.*, to the performance from the client’s point of view. For this reason we call our new performance measure the trading performance.

COMPUTATION WITH PURCHASES AND SALES ONLY

The computation of the performance attribution combines the techniques of the “Turnover Performance” of Cantaluppi (2013) and the “Contribution of Initial Holdings and Transactions to Performance” of Cantaluppi (2014), both in a slightly adjusted form, which we will explain later. The computation is first computed as a nominal performance, and the IRR performance is done later on. We will show that the difference between the contribution of the transaction to performance and its turnover performance can be attributed to the selection capabilities of the asset manager.

Let $(0, T)$ be the performance interval, q_t^i the quantity of security i in the portfolio at time t and p_t^i the price of security i at time t . Let c_t be the cash position at time t . In a first step, consider that there are no corporate actions over the whole interval $(0, T)$. We will show later how to adjust the computations in order to take corporate actions and other special transactions into account.

As seen in Cantaluppi (2014), we define the contribution of the initial holding in security i to the performance C_H^i as the difference between the value of the initial holding in this security at time T and at time 0 , *i.e.*,

$$C_H^i = q_0^i p_T^i - q_0^i p_0^i$$

This is nothing but the nominal performance of the initial holding in security i if no transaction is made on this security, *i.e.*, a “buy and hold” analysis for security i .

We now recall the computation of the contribution of transactions to the performance and their turnover performance. For reasons of simplicity, we will assume that there are no transaction fees.

First, consider a purchase transaction occurring at time t . A quantity q_t^i of security i is bought at time t for a price of p_t^i per piece, *i.e.*, for a total amount of $q_t^i p_t^i$. If we kept these securities until time T they would have

a value of $q_t^i p_T^i$. We define the contribution to performance C_x of the purchase transaction as the difference between these two values, *i.e.*,

$$q_t^i p_T^i - q_t^i p_t^i$$

The contribution to performance of the transaction is, therefore, the nominal performance of the bought securities. This contribution to performance can be rewritten as

$$C_x = q_t^i p_t^i \left(\frac{p_T^i}{p_t^i} - 1 \right)$$

If the cash account is in foreign currency, we have to take into account that its value will vary over time and this variation adds to the contribution of the transaction to performance and we, therefore, have

$$C_x = q_t^i p_t^i \left(\frac{p_T^i}{p_t^i} - 1 \right) - q_t^i p_t^i \left(\frac{p_T^c}{p_t^c} - 1 \right)$$

where p_T^c is the price of one unit of cash at time T (exchange rate).

Now let security i be in asset class A and define b_t^A the value of the benchmark for asset class A at time t . Also, define b_t^c the value of the benchmark for cash at time t . The turnover performance A_x of the transaction is then defined as in Cantaluppi (2013) as

$$A_x = q_t^i p_t^i \frac{b_T^A}{b_t^A} - q_t^i p_t^i \frac{b_T^c}{b_t^c}$$

Combining these equations, we define

$$S_x = C_x - A_x = q_t^i p_t^i \left(\frac{p_T^i}{p_t^i} - \frac{b_T^A}{b_t^A} \right) - q_t^i p_t^i \left(\frac{p_T^c}{p_t^c} - \frac{b_T^c}{b_t^c} \right)$$

We can consider the value S_x as the selectivity component of the transactions since it is the difference between the effective return and the benchmark return of the investment in the security i minus the difference between the effective return and the benchmark return of cash retrieved. A sale transaction would produce similar results with a negative sign, again providing an intuitive justification for the definition of the selection component of the transaction.

If a sale and a purchase are made the same day for the same amount for two securities in the same asset class, it is easy to see that the turnover performances of the two transactions sum to zero. The selectivity compo-

nents of the two transactions sum to a value which is the difference of nominal returns between the bought and the sold securities, corresponding to what we expect from a selectivity measure.

COMPUTATION WITH CORPORATE ACTIONS

To accommodate for corporate actions during the interval $(0, T)$, we adjust the end price p_T^i of the security for each transaction occurring before a corporate action for this security. If a purchase, for example, occurs at a time t preceding a dividend payment of d^i , the end price for the calculation of the nominal performance of this purchase will be taken as $p_T^i + d^i$. The dividend transaction itself will then be ignored in the computation of the nominal performance.

The principle for the calculation of the contributions is the same for complex corporate actions as in the simpler cases described above. This means that the computation of the contributions concerning security i necessitates the price of security i at time T . If there has been a corporate action for security i during the interval $(0, T)$ the price of security i at time T is either not known because security i does not exist anymore (*e.g.*, after a merge), or the price of security i does not reflect the reality because security i at time T does not represent the same entity (*e.g.*, after a split or a spin off). We will, therefore, replace the price of security i at time T by the weighted prices at time T of the securities arising from corporate actions, the weights being given by the corporate action. If the corporate action includes a cash component it will be handled the same way the dividend payment is handled in the case described above; *i.e.*, it increases the contributions of the initial holdings and purchases prior to the corporate action date and decreases the contributions of sales prior to the corporate action date.

Let us formalize this process. Assume that we have a complex corporate action for security i at time t . One unit of security i gives c units of cash and units of security j for a given set of securities j that might include security i . The price p_T^i of security i at time T is given by the expression

$$\sum_j q_j p_T^j$$

Of course one of the securities j could itself have a cor-

porate action between the time of the corporate action of i and time T . This problem can be easily solved by working the corporate actions backward from time T to time 0 . As mentioned above the cash part of the corporate action is handled separately, similarly to a dividend payment.

Now that we have “prices” at time T for all securities in the portfolio at time T , we can compute the contributions of initial holdings and transactions to the nominal performance as we did for the simpler cases.

This procedure is consistent with the choice of benchmarks with reinvested dividends (total return benchmarks), which is the standard choice for performance benchmarks. The turnover component of the attribution can therefore be computed as usual, without special consideration for dividends and other corporate actions.

RETURN TIME SERIES

So far we have computed the nominal contributions to performance of the initial holdings and of the transactions. However, we would like to decompose the MWR performance of the investment and not its nominal performance. We will now see how the two measures relate and how to transform the nominal performance into the MWR performance of the investment. We will compute the MWR performance as its internal rate of return (IRR). Let V_t be the value of the investment at time t and let C_F^j be the j -th cash flow occurring at time t_j . Then, if r is the internal rate of return of the investment, we have

$$V_T = V_0(1+r)^T + \sum_j C_F^j (1+r)^{T-t_j}$$

On the other hand, the nominal contributions to performance have been so defined that the value equation

$$V_T = V_0 + \sum_i C_H^i + \sum_k C_X^k + \sum_j C_F^j \text{ equation}$$

holds (Cantaluppi (2014)). The value equation simply states that the end value of the investment equals the beginning value of the investment plus the contributions

$$V_0((1+r)^T - 1) + \sum_j C_F^j ((1+r)^{T-t_j} - 1) = \sum_i C_H^i + \sum_k C_X^k$$

of the initial holdings, of the transactions and of the capital flows. We, therefore, have

i.e., the sum of the contributions to performance of the initial holdings and the transactions (right hand side) is equal to the combined nominal return of the initial investment and of the cash flows (left hand side). It is, therefore, legitimate to scale the nominal contributions of the initial holdings, and of the transactions into percentage contributions, so that they add up to the internal rate of return r . By doing so, we not only know the money-weighted rate of return of the portfolio as computed by the IRR, we also know where it comes from since we know the contribution of each initial holding and each transaction to this result. We can, therefore, cumulate the contributions up to any time t and present the final result not simply as a return, but as a time series of returns.

The value of the time series at time 0 is the contribution to performance of the initial holdings, and the value at time T is the total performance for the given period. The change of value of the time series at a given date is the sum of the contributions to return for all transactions at that date.

The resulting graph allows a visualization of the periods with favorable and unfavorable transactions. The sharp changes of the curve readily underline the important transactions of groups of transactions, which can then be analyzed in detail.

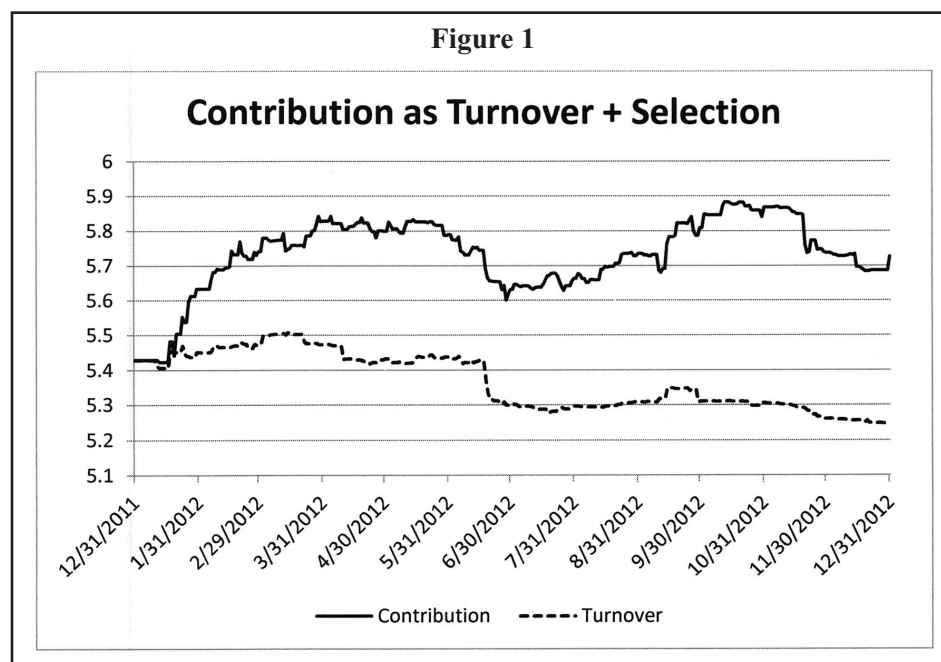
PROPERTIES OF THE NEW PERFORMANCE ATTRIBUTION MODEL

We restate the main properties of this new trading performance attribution model. We emphasize that the underlying assumptions are quite different from those of the Brinson model. Both methods give results that cannot be directly compared.

1. This is an attribution model of the MWR performance, it is not the first such model (see, for example, Illmer & Marty (2003)). The justification for an MWR attribution can be stated by a desire to answer the question, “What has the manager done for me?” A more technical justification is presented by Illmer & Marty: “However, there is also a need for calculating and decomposing the MWR because it is the MWR which covers the timing effect of cash flows

solute profit and loss of a portfolio.”

2. The investment strategy does not play a central role as in the Brinson model. In fact, the strategy weights of the asset classes are irrelevant for the attribution. This results from three considerations: the strategy weights are always associated with minima and maxima that give latitude to the asset manager, the neutral asset manager does not constantly rebalance the asset classes according to the strategy weights, and finally, the weights of the asset classes change over time without the involvement of the asset manager, reflecting the different tendencies of the market for the different asset classes. “Doing nothing” is considered a neutral policy in our model as opposed to active management in the Brinson model. However, the benchmarks of the asset classes are also important in this new model as they are the basis for the computation of turnover performance. The asset classes of this new performance attribution model are, therefore, merely groups to which the return is attributed. They can be industry sectors, geographic regions, etc.
3. The contribution of each transaction to performance, with decomposition into turnover and selection, is a major improvement over other attribution models. We have seen clients analyze thousands of transactions in order to study the decisions of the asset manager, mostly without success. The decisions of the asset managers are rarely reflected in one single transaction. They are rather reflected in many transactions, not necessarily occurring at the same time. The attribution at the level of asset classes, or any group of assets, can therefore be computed by aggregation of the contributions of initial holdings and transactions.
4. The presentation of the results as a graph highlights transactions or groups of transactions that have been decisively influenced the performance. Moreover, the contribution of these transactions is plainly split into a turnover and a selection component, revealing the skills of the asset manager.
5. The whole computation necessitates the valuation of the portfolio at the beginning and at the end of the performance period only. No weights are needed during the performance interval, meaning that no prices are needed during the performance interval, except those of the benchmarks. On the other hand, corporate action data are needed to adjust the end prices for each corporate action. Prices for the processing of transactions are given by the transaction data themselves.
6. The calculation of the attribution through the computation of the nominal performance, consistent



with the MWR performance, eliminates the need for an interaction component that cannot be attributed to any decision or skill of the asset manager.

The different assumptions underlying this new model and the Brinson model suggest that one model does not constitute an alternative or a replacement for the other, but a complement. These assumptions also suggest that the Brinson model might be a better analysis tool for a top-down approach to the asset management, and our new model, a better analysis tool for a bottom-up approach to asset management.

REAL LIFE EXAMPLE

We now present the result of the trading performance for a real world case: an internationally diversified pension fund portfolio for the year 2012. The following graph shows the time series of the contributions to performance, divided into a turnover and a selection component. The value of the time series at 2011-12-31 is the contribution to performance of the initial holdings, *i.e.*, the “do nothing” performance. The value of the time series at 2012-12-31 is the total MWR performance for the year 2012. The value of the time series at any date in between is the contribution to performance of the initial holdings and all transactions up to that date. The “do nothing” performance is 5.43% and the MWR performance is 5.72%, meaning that the active decisions of the asset manager during 2012 have improved the performance of the portfolio by 0.29%, compared with a scheme where no trades would have been completed. This out-performance can be in turn decomposed into a turnover component of -0.18% and a selection component of 0.47 percent. The solid line of the graph is the contribution to performance, and the dashed line is the turnover performance, the difference being the selection component.

The contributions to performance are computed at the level of single transactions. There are no obvious jumps in the lines of the graph related to one transaction or even to the transactions of one day. The contribution to performance of each transaction is relatively small and only the cumulating of these contributions for a relatively large number of transactions can show a trend. For example, the transactions between January 15 and February 19 show a spectacular increase in performance, and the transactions between May 29 and June 26 show a decrease in performance. This confirms the fact

that the analysis of large transactions usually does not produce any valuable result. Large transactions provide no answer because the manager does not apply her tactical change with one large transaction but with many smaller transactions that can be spread over a period of several days.

CONCLUSION

The Brinson model provides an extremely valuable tool to analyze the performance of an investment by evaluating the deviations from the investment strategy and from the benchmark. But the deviations from the investment strategy do not necessarily derive from direct investment decisions on the part of the asset manager. Investors and counselors therefore tend to directly analyze the active decisions of the asset manager, *i.e.*, the resulting transactions, especially the purchases and sales. This is a Sisyphus¹ task since the number of transactions can be very large and the decisions of the asset manager are rarely implemented with one single transaction, but with many transactions, possibly spread over a long period of time.

The trading performance presented in this article is a possible answer to the arduous question of analyzing the performance at the transaction level. The problem caused by the sheer number of transactions remain, but the trading performance assigns a performance contribution, divided into a turnover and a selection component to each one of these transactions. This computation can provide the basis for an analysis tool that can search for groups of transactions having substantially contributed, positively or negatively, to the performance of the investment.

Last, but not least, we discovered during the discussion with an asset manager that she uses a simplified version of the analysis presented above for evaluating her decisions regarding her stock trades for the past month. She compares the value of her portfolio at the end of the month with the value that her portfolio at the beginning of the month would have at the end of the month, manually adjusting this value to take into account the corporate actions that occurred during the month. This is exactly the “do nothing” value of the portfolio. This real world behavior convinced us that the trading performance has a great practical value and is not only a nice looking theory.

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ENDNOTES

¹ King Sisyphus was made to endlessly roll a huge boulder up a steep hill. The maddening nature of the punishment was reserved for King Sisyphus due to his hubristic belief that his cleverness surpassed that of Zeus himself. Zeus accordingly displayed his own cleverness by enchanting the boulder into rolling away from King Sisyphus before he reached the top which ended up consigning Sisyphus to an eternity of useless efforts and unending frustration. (Source: Wikipedia)